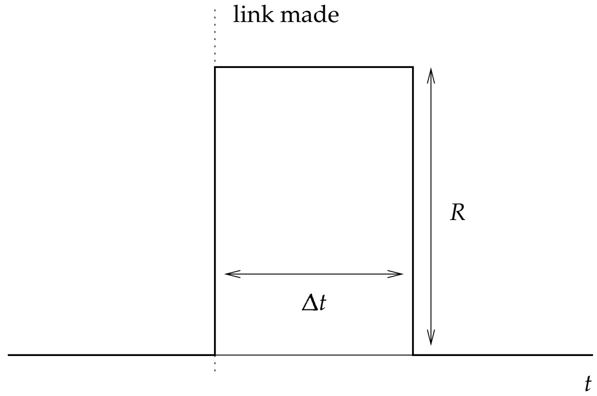
Web traffic spikes

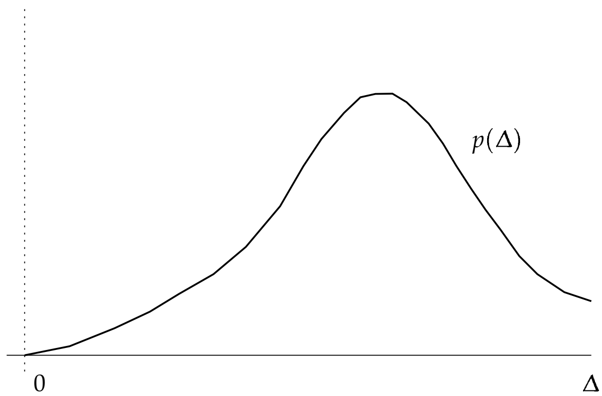
Suppose that N people visit a website A every Δ seconds. Although each person looks at the website at fixed intervals, they start at different times. Assume these start times are uniformly distributed. At some time $t = 0$, the website posts a link to another website, B. Assume that visitors to A at times $t \geq 0$ will follow the link the first time they see it.

Clearly website B will see continuous traffic at a constant rate $R = N/\Delta$ (in hits per second) for $0 \leq t \leq \Delta$, since visitors from the first website will arrive from the moment after the link is posted (corresponding to users who last checked the website at time $t = -\Delta$) until time $t = \Delta$ (corresponding to users who last checked the website a moment before the link was posted):



Call this function $Rh(t/\Delta)$, where $h(x)$ is the function which is 1 for $0 \leq x \leq 1$ and 0 elsewhere.

Realistically, all the users of a website do not check it at equal fixed intervals. Instead consider a population N_0 of users, each of whom has a different check interval Δ. The Δ are distributed according to $p(\Delta)$, so that there are $N_0 p(\Delta) d\Delta$ users who check the website at intervals between Δ and $\Delta + d\Delta$:



(Note that $p(0) = 0$.) The traffic experienced by B is the sum of a set of functions like $h(t)$. There are $N_0 p(\Delta) d\Delta$ users who check site A at intervals $[\Delta, \Delta + d\Delta]$, and these users will give rise to traffic $N_0 h(t/\Delta) p(\Delta) d\Delta/\Delta$. The total traffic $r(t)$ experienced by B at $t \geq 0$ is then given by,

$$r(t) = N_0 \int_0^\infty \frac{h(t/\Delta)p(\Delta)d\Delta}{\Delta}.$$

$h(t/\Delta)$ is a function which is 1 when $\Delta > t$ and 0 when $0 \leq \Delta \leq t$. So,

$$r(t) = N_0 \int_t^\infty \frac{p(\Delta)d\Delta}{\Delta}.$$

Notice that $r(t)$ is a strictly decreasing function of t, so the traffic spikes at $t = 0$ and declines after that. As an analytic example, suppose that $p(\Delta) = 1/(b-a)$ for $a < \Delta < b$, and 0 elsewhere. This yields,

$$\frac{(b-a)}{N_0} r(t) = \begin{cases} \log(a/b) & t < a \\ \log(t/b) & a \leq t < b \\ 0 & b \leq t \end{cases}$$

Sketching the form of $r(t)$, we see something like: